Simulation of channel segregation using a two-phase columnar solidification model – Part II: Mechanism and parameter study

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1. Introduction

Channel segregates, e.g. A-segregates in large steel ingots and freckles in unidirectionally solidified castings, are the consequence of thermal–solutal convection and the resulting flow interaction with the solidifying mushy zone [1–6]. Due to the engineering significance of alloy quality and durability, the formation mechanism of channel segregates has been the central focus of much theoretical and experimental research for many decades [4–12]. As large-scale computational power continues to become more accessible and affordable, numerical studies have also become a significant contributor to the investigation of channel segregates in the last decade [3,12–17].

The formation of channel segregates in a casting is dependent on the flow conditions present during solidification. The initial formation of channel segregates is believed to be the result of convection instability in the mushy zone near the primary dendrite tips [6]. For an alloy with a partition coefficient less than one (k < 1), the interdendritic melt becomes enriched with solute elements, which causes a change in the local melt density. Thus, the density of the interdendritic melt is a function of both concentration and temperature. Depending on the thermal and solutal expansion coefficients and the solidification direction, varying thermo-solutal convection patterns in the mushy zone can develop as shown in Fig. 1. In the majority of engineering casting situations convection usually occurs as in Fig. 1b–d.

According to Worster [7] the onset of convection instability and channel formation can be analyzed in terms of the mushy zone Rayleigh number $R_a$, defined as the ratio of the thermo-solutal buoyancy force to the opposing friction force associated with the mush permeability. Beckermann and co-workers [12] have extended the previous definition of $R_a$ by incorporating a more precise consideration of the solidification kinetics and the variable permeability of the mush:

$$R_a(h) = \frac{((\rho_0 - \rho_1)/\rho_0) \eta K h}{\alpha v}.$$  (1)

In this expression the $R_a$ number is a function of the position $h$ in the mushy zone, where $h$ measures the depth of the mush from the primary dendrite tip, shown in Fig. 1b. The $\alpha$ and $v$ are thermal

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diffusivity and kinematic viscosity, $\rho_0$ is the reference density corresponding to the liquidus temperature and $R$ is the mean permeability. It was found that if the maximum $R_s$ in the mushy zone exceeded a critical number, 0.25, freckles tend to form.

Given that $R_s$ varies significantly with thermal parameters such as the temperature gradient $G$ and the solidification velocity $R$, Beckermann’s definition of the $R_s$ criterion can also be approximated as

$$\frac{1}{R^2 G^2} < \text{constant}. \tag{2}$$

where $n'$ and $n''$ are 1/2 and 1/5, respectively. When the above condition is fulfilled, channel segregates will not form. Other experimental researchers have also proposed similar criteria, with varying exponents and constants. For example, in an NH$_4$Cl–H$_2$O system, the exponents $n'$ and $n''$ were found to be 1 and 1 [11]; Pollock and Murphy suggested 1/2 and 1/4 for the Ni-based superalloys [18]; and Suzuki and Miyamoto used 2.1 and 1 for large steel ingots [8,9]. As $G$ and $R$ are important, but not the only factors influencing channel segregation, the above discrepancy suggested further investigation as worthwhile. The onset of convection instability is not a sufficient condition to ensure the formation of channels; the channels must be stabilized by the resulting flow-solidification interaction. The coupling between the interdendritic flow and solidification is a key factor for channel growth. Solidification of the mushy zone can be accelerated at one location, or suppressed (even remelting) at another location by the flow [4,5,12]. For the unidirectional solidification case in Fig. 1b, the segregated melt is lighter, tends to rise and induces a convection cell. The rising melt, bringing the segregated melt upwards, suppresses solidification and/or encourages localized remelting such that a pencil-like plume (chimney) forms, leading to the formation of freckles as observed in many super-alloy castings. For the lateral solidification cases of Fig. 1c and d, flow perturbation can occur at different positions (height). However, the flow perturbation can only create stable channels in some preferential regions. Meherabian and co-authors proposed an analytical correlation between the solidification rate and the flow velocity, $\bar{u}_s$, [4,5] based on Flemings’ local solute redistribution equation [1].

$$\frac{\partial f_C}{\partial T} = -\left(\frac{1 - \beta \frac{f_C}{f_C'}}{1 - k \frac{f_C'}{f_C}}\right) \left(1 + \frac{\bar{u}_S \cdot \nabla C}{T}\right). \tag{3}$$

Here $\beta = (\rho_1 - \rho_2)/\rho_1$, the solidification shrinkage, and $c_l'$ is the thermodynamic equilibrium concentration, assumed to be equal to the local melt concentration, $c_l$. The sign of the first term in parenthesis on the right hand side of Eq. (3) is always negative regardless of $k$, and the sign of the cooling rate $T$ is always negative. Therefore, the sign of $\partial f_C/\partial T$, determining the acceleration or suppression (even remelting) of solidification, depends on the sign and magnitude of the term $\bar{u}_S \cdot \nabla C$. This analysis predicts that a stable channel will form when $\bar{u}_S \cdot \nabla C > |T|$, which causes $\partial f_C/\partial T$ to be negative.

Table 1

<table>
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<tr>
<th>Simulation case</th>
<th>$\bar{u}_S$ (mm)</th>
<th>$\beta$</th>
<th>$R_{Max}^{\text{Max}}$</th>
<th>Channel or not</th>
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<td>-1.06</td>
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</tr>
<tr>
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<tr>
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<td>-0.53</td>
<td>13.22</td>
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</table>

*Maximum Rayleigh number ($R_{Max}^{\text{Max}}$) being reached during the initial stage (10 s) of solidification.
negative. In other words, a sufficiently strong interdendritic flow in the direction of the temperature gradient will cause remelting of dendrites and an open channel will form. If $\bar{u} \cdot \nabla T < 0$, i.e. flow is in the direction opposite to the temperature gradient and $\partial f_i / \partial T$ is positive, hence there is no possibility for stable channel to form. A third, intermediate case arises if $|\bar{u}| > \bar{u} \cdot \nabla T > 0$, i.e. interdendritic flow is in the direction of temperature gradient and $\partial f_i / \partial T$ is still positive, but is reduced by the flow. In other words, solidification still proceeds with decreasing temperature, but the solidification rate is locally suppressed by the flow. In this intermediate case channels might form.

Eq. (3) can be applied to qualitatively analyze the potential formation of channel segregates. For example, in the lateral solidification case of Fig. 1c, the interdendritic melt, with a relatively higher concentration of solute element, is lighter and tends to rise and escape from the mushy zone in the upper part of the casting. The escaping melt suppresses solidification and may cause localized remelting, such that channels form, leading to the A-segregation observed in many steel ingots. Channels are not able to form in the bottom region in such a configuration. In the lateral solidification of Fig. 1d where the interdendritic melt is denser, channels form in the bottom region, as demonstrated in the current Sn–Pb benchmark (Part I).

Part II of this investigation examines the formation mechanism of channel segregates in the Sn–10 wt.% Pb benchmark using a two-phase columnar solidification model [19,20]. The current model differs from previous channel segregation models by including diffusion of the solute element in the interdendritic melt as a governing factor of solidification. Therefore, the significance of the diffusivity of the solute element is also discussed. An in-depth parameter study of the $R_a$ number, by varying the mush permeability, is presented. The benchmark studied is similar to the lateral solidification case of Fig. 1d, however the conclusions from this study can be applied to the case of Fig. 1c.

2. Growth of channels

2.1. Flow-solidification interaction

After the initial formation, a channel may either continue to grow or disappear depending on the flow-solidification interaction in the two-phase mushy zone. Following the work of Meherabian and co-authors [4], a correlation of the local solidification rate $(M_i')$ to the flow velocity $(\bar{u} \cdot \nabla T)$ is established to better capture the phenomena of channel segregation in the situation of diffusion-governed solidification. The mass and species conservation equations for the two-phase columnar solidification model are summarized in Table 1 of Part I. A detailed description of the model is presented in the authors’ previous publications [19,20]. The current model includes the following key assumptions [21]:

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![Diagram showing concentration and velocity fields](image)

**Fig. 2.** Analysis of the formation of the channel segregation at $t = 40$ s: (a) mixture concentration ($c_{mix}$) distribution in the cavity is shown in gray scale and isolines; (b) contours of the flow-solidification interaction term $(\bar{u} \cdot \nabla C_i)$ in white (positive) and black (negative) overlaid by the mass transfer rate $(M_i)$ isolines; (c) liquid volume fraction $(f_l)$ contours and $f_l$ isolines overlaid by vectors of liquid velocity and streamlines; and (d) $c_{mix}$ contour and its isolines.
The species conservation equation is written as

\[ \frac{\partial \rho_f c_i}{\partial t} + \rho_p \mathbf{u} \cdot \nabla c_i = \frac{\partial}{\partial t} \left( \rho_f c_i \right) + \nabla \cdot (\rho_f \mathbf{u} c_i) = 0. \tag{4} \]

Substituting the mass conservation equation into Eq. (4) the time derivative of the liquid concentration is

\[ \frac{\partial c_i}{\partial t} = - \frac{c_i - c^*}{f_i} \frac{\partial f_i}{\partial t} - \mathbf{u} \cdot \nabla c_i. \tag{5} \]

Taking the time derivative of \( T = T_f + mc_i \), the changing liquid interface concentration is

\[ \frac{\partial c_i^*}{\partial t} = \frac{1}{m} \frac{\partial T}{\partial t}. \tag{6} \]

Subtracting Eq. (5) from Eq. (6) results in the local rate of change of \( (c_i - c^*) \)

\[ \frac{\partial (c_i - c^*)}{\partial t} = \frac{c_i - c^*}{f_i} \frac{\partial f_i}{\partial t} + \frac{1}{m} \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla c_i. \tag{7} \]

In the left hand side (LHS) of Eq. (7), \( (c_i - c^*) \) is the driving force for solidification/melting, which governs the solidification/melting rate, \( M_{\text{sc}} \). \[19–21\].

The local solidification/melting rate is the result of three contributions, corresponding to the three right hand side (RHS) terms of Eq. (7). The first RHS term is the contribution of the solidification-induced solute enrichment of the interdendritic melt, the sign of which is always negative for solidification. The solidification rate decreases with solute enrichment of the interdendritic melt. The second RHS term is the contribution of the cooling rate, the sign of which is always positive, i.e., the solidification rate increases with enhanced cooling. These first two terms are relatively stable during solidification and are not considered critical factors in channel formation. The third RHS term in Eq. (7), \( \mathbf{u} \cdot \nabla c_i \), is a flow-solidification interaction term, which is the most critical for the formation of the channel. Depending on the interdendritic flow, the sign of this term can be positive or negative. Local solidification behavior depends on the sign of the flow-solidification interaction term. In a region where the melt flows in the same direction as the concentration gradient, the flow-solidification interaction term is positive. The local increase in fluid velocity due to a flow perturbation accelerates solidification and as a consequence of the locally accelerated solidification the flow permeability (K) becomes relatively smaller than that of neighboring zones and the interdendritic flow slows down. In other words, the increase in fluid velocity induced by a flow perturbation is throttled through the enhanced solidification. In the same sense, the local decrease in fluid velocity due to a perturbation is reinforced by the decreased solidification rate. The region with a positive flow-solidification interaction term is an accelerated solidification region where flow perturbations are throttled through the flow-solidification interaction and channels do not form.
In regions where the melt flows in the opposite direction of the concentration gradient, the flow-solidification interaction term is negative. The local increase in flow velocity due to a flow perturbation suppresses the solidification rate. This region with a relatively lower solid fraction has a larger permeability and the flow becomes stronger. In other words, the local increase in flow velocity due to a flow perturbation is reinforced by the suppressed solidification. In the same sense, a local decrease in flow velocity due to a flow perturbation is further decreased by the increased solidification rate. As a consequence of the flow-solidification interaction, the magnitude of the initial flow perturbation is increased. The region with a negative flow-solidification interaction term is a suppressed solidification region where channels form.

The above hypothesis on channel formation is verified by the current modeling results, as detailed in Figs. 2–4. The results shown here are obtained from a 2D simulation of Sn–10 wt.% Pb benchmark (0.05 \( \times \) 0.06 m \(^2\)). Heat is extracted from the vertical (right) wall, and solidification proceeds laterally from the right towards the plane of symmetry. The two-phase columnar solidification model, computational domain, material properties and process parameters, were described in Part I of this study.

At 40 s channels are observed in the right bottom region, as shown in Fig. 2a. The global flow pattern in the cavity is similar to the case of Fig. 1d, namely the solute enriched interdendritic melt is denser than the bulk melt. The interdendritic melt sinks in the mushy zone, causing a suppressed solidification zone in the lower region. A zoomed view shows the details of the channel formation in Zone 1 (near right bottom corner) in Fig. 2b–d. The suppressed solidification zones, corresponding to negative \( \mathbf{u}' \cdot \nabla c' \), are shown in black in Fig. 2b. The solidification rate in the suppressed-solidification (black) zones is significantly decreased by the flow. The solidification (mass transfer) rate, \( M_c \), in these zones is relatively small, \( \sim 20 \text{ kg m}^{-3} \text{ s}^{-1} \), in comparison with neighboring zones where \( M_c \) reaches as high as \( 120 \text{ kg m}^{-3} \text{ s}^{-1} \). The change in solidification rate due to a flow perturbation leads to the growth of the channel. As soon as a channel forms, the flow takes the path of least resistance through the channel, in Fig. 2c, hence the flow streamlines are strongly influenced by the forming channels. Inside the channels the interdendritic flow travels through the channel paths while in the immediate neighboring zones the flow is diverted almost vertically across the channel wall(s). The macrosegregation distribution \( \bar{c} \) in Fig. 2d has a similar distribution pattern to \( f \) and the flow-solidification interaction term. Throughout the solidification process, the value of \( M_c \) stays positive—indicating that the mass is always transferred from the liquid to the solid phase, and channels form without evidence of remelting.

The dynamic evolution of the channel is further analyzed in Fig. 3 by tracking the flow-solidification interaction term \( \mathbf{u}' \cdot \nabla c' \), mass transfer rate \( M_c \), liquid volume fraction \( f \) and mixture concentration \( \bar{c} \) along a vertical path in the simulation domain. The path (Path I, as marked in Fig. 2d) crosses one channel, from 0.0025 m to 0.0065 m (distance from the bottom). The formation of the channel can be analyzed by \( f \) and \( \bar{c} \) curves,
Fig. 5. Evolution of the flow-solidification interaction term, which is defined by $\vec{u}_i \cdot \nabla c_i$ and $\vec{G}$, $\overline{G}$. The curves are plotted along the path 1 as marked in Fig. 2d. (a) 5 s; (b) 7 s; (c) 9 s; (d) 11 s.

when they become arched-upwards. Generally, the curves of $\vec{u}_i \cdot \nabla c_i$ are quite similar to the curves of $M_{of}$. Fig. 3a and b, indicating a strong interaction between the flow and the mass transfer. This further verifies that among the three RHS terms of Eq. (7) the flow-solidification interaction term is the most critical in determining the variation of $M_{of}$. If the flow-solidification interaction term is not negative (e.g. before 7 s), channels are not prone to form. At 9 s, the flow-solidification interaction term stays locally negative in a somewhat oscillatory manner, the flow-solidification interaction term is not negative (e.g. before 7 s), channels are not prone to form. At 9 s, the flow-solidification interaction term is negative in the upper section of Path I – solidification is suppressed leading to a reduction of $M_{of}$. However, the arching of the $f_i$ curve, an indicator of channel formation, is not yet evident. Up to 13 s the flow-solidification interaction term stays locally negative in a somewhat oscillatory manner, the $M_{of}$ curves become concave-downwards, and the onset of channel formation can be identified from the $f_i$ curve and $c_{max}$ curve. At 15 s in the suppressed solidification region $M_{of}$ is only 9 kg m$^{-3}$ s$^{-1}$, about 200 kg m$^{-3}$ s$^{-1}$ smaller than the neighboring regions. As the difference in solidification rate between the suppressed solidification area and neighboring regions becomes greater, the channel becomes larger and more stable. Again, during the entire solidification process, $M_{of}$ is never negative-formation of the channel appears to be entirely due to the change in the solidification rate caused by flow instability.

As proof of the aforementioned hypothesis, that channels cannot form in the solidification-accelerated region, the solidification process in the accelerated solidification region is also examined, as shown in Fig. 4. The flow-solidification interaction term $\vec{u}_i \cdot \nabla c_i$ is positive everywhere in Zone 2. Any increase in flow due to local flow perturbations will be throttled by the accelerated solidification. The $f_i$ isolines and the flow streamlines are smooth and undisturbed, thus, channels do not appear in this region.

Recently, Sawada and co-authors [22] studied the mechanism of channel segregation in vertical directional solidification with a Pb–10 wt.% Sn alloy, and reported a similar conclusion regarding the accelerated and suppressed solidification zones.

2.2. Significance of the finite diffusion in the interdendritic melt

Eqs. (3) and (7) differ in the treatment of diffusion of the solute element in the interdendritic melt. Eq. (3) is derived on the assumption of infinite diffusion of the solute element in the interdendritic melt while Eq. (7) includes this diffusion. When an assumption of infinite diffusion in the interdendritic melt is applicable (i.e. $c_i = c'_i$), one can use the definition of the flow-solidification interaction term $\vec{u}_i \cdot \nabla c_i$ instead of $u_i \cdot \nabla c_i$ to analyze channel formation. For many engineering cases of solidification, infinite diffusion in the interdendritic melt represents a reasonable approximation and the term $\vec{u}_i \cdot \nabla c_i$ has been successfully used to analyze channel formation [4,5]. However, when the influence of the interdendritic diffusion is significant, an analysis based on the term $u_i \cdot \nabla c_i$ is more reliable.

Fig. 5 shows the evolution of the flow-solidification term, which was defined in two ways, i.e. $\vec{u}_i \cdot \nabla c_i$ and $\vec{G}$, $\overline{G}$, along the same path. As expected, the evolution of the $\vec{u}_i \cdot \nabla c_i$ curves and $\vec{G}$, $\overline{G}$ curves show quite similar behavior. Small differences exist in the curves, particularly in the lower region of the benchmark, however the oscillation frequencies between the positive and negative values show the same trend. This indicates that the flow-solidification interaction term ($u_i \cdot \nabla c_i$), based on the infinite diffusion assumption, is a good approximation for a qualitative investigation on the formation of the channel segregation for the current bench-
mark. To verify the significance of solute diffusion in the interdendritic melt, further simulations were made by varying $D'$ from (a) $1.0 \times 10^{-7}$, (b) $1.0 \times 10^{-8}$, (c) $1.0 \times 10^{-9}$ m$^2$ s$^{-1}$. As shown in Fig. 6, the tendency for channel segregation to occur (the amount of channels, the area where the channels occur, and the severity of segregation) increases as $D'$ increases. The case with $D' = 1.0 \times 10^{-7}$ is closest to the assumption of infinite diffusion, where channel segregation is slightly overestimated. When the diffusivity is very small, $1.0 \times 10^{-9}$, channel formation is inhibited. This indicates that the validity of $u' \cdot \nabla c_{\text{mix}}$ for the investigation of channel formation is limited to alloys with large solute diffusivity in the interdendritic melt.

3. Onset of channels

3.1. Parameter study

As previously studied, the onset of channel formation can be characterized by the Rayleigh number [7,12], i.e. the ratio of the thermo-solutal buoyancy force to the friction force associated with mushy zone permeability. The mushy zone permeability is proportional to the square of secondary dendrite arm spacing, $K \propto L_2^2$, and the key parameters for the buoyancy force are the expansion coefficients $\beta_c$ and $\beta_T$.

Thus, the sensitivity of the channel segregation to Rayleigh number is investigated by varying $L_2$ and $\beta_c$, as listed in Table 1. All other physical and process parameters are kept constant.

Figs. 7 and 8 compare the simulation results of four cases by varying $L_2$. As $L_2$ increases resistance to the interdendritic flow decreases and heat and mass transfer are enhanced. As shown by the isotherms in Fig. 7 the heat extraction in Case 13 (large $L_2$) appears to be faster than that in Case 2 (small $L_2$). At $t = 150$ s the temperature near the plane of symmetry of Case 13 is lower than that of Case 2, and the isotherms move slightly faster toward the plane of symmetry as $L_2$ increases.

The influence of $L_2$ on $f'$ and $c_{\text{mix}}$ is more evident than the influence of $L_2$ on heat transfer. When $L_2$ is small (<100 μm) the friction resistance of the dendrites is high enough to maintain a stable interdendritic liquid flow; channels and channel segregation are not observed, as shown in Figs. 7a and 8a. Channels and channel segregation occur only when $L_2$ is sufficiently large (>260 μm), as shown in Figs. 7b–d and 8b–d. The number of the channels found in the calculation domain is dependent on $L_2$. For $L_2$ equal...
to 260, 455, and 650 µm, the number of channels formed is 4, 7 and 11, respectively. The length of the channels is also influenced by $\lambda_2$. With a larger $\lambda_2$, the channels extend (penetrate) deeper into the mushy zone. For Case 13 ($\lambda_2 = 650$ µm) the channel segregation zone occupies more than half of the mold cavity. Based on these results, it can be concluded that a large Rayleigh number promotes channel formation.

Examining the global maximum $c_{\text{mix}}$, which occurs in the lower left (near the plane of symmetry), and the global minimum $c_{\text{mix}}$, which occurs in the upper right region near the casting surface, shows that the minimum $c_{\text{mix}}$ and maximum $c_{\text{mix}}$ are also strongly influenced by $\lambda_2$ (Fig. 9). Two situations spanning the range of $\lambda_2$ must be distinguished. Firstly, when $\lambda_2$ is small (<130 µm) when very few or no channels appear, the maximum $c_{\text{mix}}$ increases while the minimum $c_{\text{mix}}$ decreases with increasing $\lambda_2$. This is due to the significantly enhanced global flow in the mold cavity, caused by the increased permeability of the mushy zone. A stronger flow means enhanced transport of segregated interdendritic melt, hence stronger macrosegregation. Secondly, when $\lambda_2$ is large (≥195 µm) and a greater number of channels appear, both maximum $c_{\text{mix}}$ and minimum $c_{\text{mix}}$ decrease with increasing $\lambda_2$. The reason for the decrease of the global maximum $c_{\text{mix}}$ is due to the entrapment of the solute element by the segregation in the channels. The minimum $c_{\text{mix}}$ occurs in the upper right region near the casting surface, an area less influenced by the channel formation. Therefore, the minimum $c_{\text{mix}}$ continues to decrease with the enhanced global flow intensity in the mold cavity.

To further quantify channel segregation and characterize channel features the following parameters are defined: maximum channel length ($l$), channel space ($d$), and channel inclination angle ($h$), as schematically shown in Fig. 10. The severity of segregation across a channel is evaluated by a channel segregation severity index ($\Gamma$), defined as

$$\Gamma = \frac{c_{\text{mix}}^\text{max} - c_{\text{mix}}^\text{min}}{c_0} \times 100\%,$$

where $c_{\text{mix}}^\text{max}$ is the local maximum of $c_{\text{mix}}$ within the channel, $c_{\text{mix}}^\text{min}$ is the local minimum of $c_{\text{mix}}$ between two neighboring channels and $c_0$ is the nominal concentration. The local maximum and minimum concentrations used in Eq. (9) are distinguished from the global maximum and minimum concentrations discussed in the previous section. Determination of the segregation parameters ($l$, $d$ and $h$) from the current simulation results, for instance in Fig. 8, is done by a visual measurement. The values of $d$ and $h$ are averaged for each simulation case. The current discussion serves only as qualitative evaluation of the dependency of the segregation parameters on $\lambda_2$. 

Fig. 7. Influence of $\lambda_2$ on $f'$ and $T$ at $t = 150$ s, where $f'$ is shown in gray scale (dark for 1 and light for 0) and $T$ is shown in isothermals. (a) Case 2: $\lambda_2 = 65$ µm; (b) Case 10: $\lambda_2 = 260$ µm; (c) Case 12: $\lambda_2 = 455$ µm; (d) Case 13: $\lambda_2 = 650$ µm.
The influence of $k_2$ on the above segregation parameters is shown in Figs. 11 and 12. The channel spacing, $d$, decreases and the maximum channel length, $l$, increases with increasing $k_2$. The channel inclination angle increases with increasing $k_2$, while the channel segregation severity index, $I$, initially increases and then decreases. Thus, channel segregation tends to occur for the cases of large $k_2$, but the severity of segregation is reduced with further increase of $k_2$. Precise evaluation of the above dependencies is out of the scope of the current study and requires further investigations.

### 3.2. Local Rayleigh number

The local $R_A$ number in the mushy zone (Eq. (1)) and its correlation to the formation of channel segregation are examined via

![Fig. 8](image_url)

**Fig. 8.** Influence of $k_2$ on $c_{\text{mix}}$, shown in gray scale and isolines ($t = 150$ s). (a) Case 2: $k_2 = 65 \mu m$; (b) Case 10: $k_2 = 260 \mu m$; (c) Case 12: $k_2 = 455 \mu m$; (d) Case 13: $k_2 = 650 \mu m$.

![Fig. 9](image_url)

**Fig. 9.** The influence of $k_2$ on the global maximum and the minimum value of mixture concentration ($c_{\text{mix}}$) in the computational domain.

![Fig. 10](image_url)

**Fig. 10.** Schematic definition of channel segregation parameters. The shaded regions indicate the channels.
The $h$ in Eq. (1) is estimated by $(T_{\text{liquidus}} - T)/G$, and the average permeability $K$ is replaced by the local $K$. Distribution of the $Ra$ number in the mushy zone is shown in Fig. 13. The $Ra$ number initially increases in the mushy zone with increasing distance from the liquidus isotherm ($h$), reaching its maximum at a position very close to the solidification front (liquidus isotherm), where the solid fraction is about 0.03. As $h$ further increases the $Ra$ number decreases in the deep mushy zone, implying that the onset of flow instability may occur very close to the solidification front in the mushy zone. If there is any possibility for a channel to form, it will form in the front of the mushy zone. Upon initial channel forma-

![Fig. 11. Influence of $\lambda_2$ on channel spacing, $\delta$, and maximum channel length, $l$.](image1)

![Fig. 12. Influence of $\lambda_2$ on channel inclination angle, $\theta$, and channel segregation severity index, $\Gamma$.](image2)

![Fig. 13. $Ra$ number distribution in the solidifying mushy zone: (a) at 2 s; (b) at 5 s; (c) at 10 s; (d) 15 s. The $Ra$ number distribution is shown in gray scale, overlaid with the liquid volume fraction $f$. The inserted diagrams show the $Ra$ number distribution profiles along the marked horizontal lines. This simulation is performed with $\lambda_2 = 455 \mu m$.](image3)
tion, the channel continues to ‘grow’ via the flow-solidification interaction. This ‘growth’ does not occur by re-melting of the already-solidified phase, it is due to the change of the solidification rate in the immediate neighboring regions.

Uncertainty over a precise definition of a critical $R$ number for channel formation has prevented us from using this criterion to predict channel formation directly. Table 1 summarizes the maximum Rayleigh number ($R_{\text{Max}}$) of each simulation case during initial stage of solidification. This $R_{\text{Max}}$ number can be used as an indicator to analyze the tendency of channel formation. The larger the $R_{\text{Max}}$, the more likely a channel will form. If $R_{\text{Max}}$ is correlated with the event of channel formation (yes or no), it is found that the cases with channel segregation must have a $R_{\text{Max}}$ larger than a critical value. This critical value is likely to fall in a range between 0.12 and 0.24, which is coincidently similar to the value as suggested by Beckermann and co-authors (0.25) [12], although here a different casting configuration and alloy are used. Normally, this critical number is dependent on the alloy even on the casting configuration. A more precise critical $R_{\text{Max}}$ number could be identified through additional studies of Case 4 and Case 5, however this would not be entirely straightforward. Case 4 without channels and Case 5 with clearly identified channels can only be visually distinguished; these differences cannot, under the current analysis, be quantitatively expressed. Secondly, the maximum $R_{\text{Max}}$ number recorded in Table 1 does not represent the value at the exact position and moment of channel formation. This value cannot be used as criterion to predict the position of the channel initialization. Additionally, as previously discussed (Section 2.1), the flow-solidification interaction plays a significant role in the channel formation as well. In the upper-right region of the benchmark where the flow-solidification interaction term $\dot{u}_I \cdot \nabla C_I$ is positive, a channel would not form even the critical $R_{\text{Max}}$ number is reached. Therefore, both the $R_{\text{Max}}$ number and the flow-solidification interaction term $\Pi \cdot \nabla C$ should be considered when analyzing the formation mechanism of the channel segregation.

4. Conclusions

In Part II of this two-part investigation the mechanism leading to the formation of channel segregation in a laterally solidified Sn–10 wt.% Pb benchmark was numerically studied. Both the initiation and growth of the channels were examined. The former is caused by flow perturbations in the mushy zone, which has been characterized by a mushy zone Rayleigh ($R$) number [6,7,12] and the latter is the result of flow-solidification interactions [4,5].

The parameter studies have verified the use of the $R$ number as a qualitative indicator for initialization of channel segregation. The cases with high $R$ number, obtained by increasing the secondary dendrite arm spacing $\lambda_2$ and the solutal expansion coefficient $\beta_L$, are prone to channel segregation. A further step is taken to correlate the maximum Rayleigh number ($R_{\text{Max}}$) with the event of channel formation. For cases with channel segregation $R_{\text{Max}}$ must be larger than a critical value, determined to be in a range between 0.12 and 0.24. Coincidently, this $R_{\text{Max}}$ is similar to the value as found by Beckermann and co-authors (0.25) [12]. The above critical value of $R_{\text{Max}}$ cannot be used for firm quantitative prediction of channel formation as it does not represent the value at the exact position and moment of channel formation. Further studies are needed to identify a well-defined critical $R_{\text{Max}}$. The $R$ criterion alone is not a sufficient condition to predict channel segregation; the growth and stability of the channels must be determined by the resulting interdendritic flow-solidification interaction. With the assumption of the infinite solute diffusion in the interdendritic melt, Mehendarian and co-authors proposed a flow-solidification interaction term $\dot{u}_I \cdot \nabla C_I$ to qualitatively analyze channel formation [1,4,5]. The justification of using this term when solute diffusivities in the interdendritic melt are large is verified by the current model. The reported numerical parameter studies have demonstrated the influence of solute diffusivity in channel formation. For low solute diffusivity, the definition of the flow-solidification interaction term $\Pi \cdot \nabla C$ should be used to avoid overestimation of channel segregation by the assumption of the infinite diffusion.

The sign of the flow-solidification interaction term, $\Pi \cdot \nabla C$, can be used to distinguish two solidification regions: a suppressed solidification region, where the sign of the flow-solidification interaction term is negative (flow is in the opposite direction of the liquid concentration gradient); and an accelerated solidification region, where the sign of the flow-solidification interaction term is positive (flow is in the same direction of the liquid concentration gradient). Channels can only occur in the suppressed solidification region, where an increase in local flow caused by a flow perturbation is reinforced by the resulting suppressed solidification. In this situation the flow becomes unstable and channels continue to grow. Channels do not form in the accelerated solidification region, where an increase in local flow intensity due to a flow perturbation is throttled by the resulting accelerated solidification. In this latter case flow perturbation is stabilized by flow-solidification interactions.

In the current benchmark simulation, remelting does not occur during channel formation, indicating that remelting is not a necessary condition for channel formation, which confirms a recent finding of Sawada et al. [22].

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