Supplementary Information: "Computation of VOF interface area"

This document is a supplementary information for the paper entitled "A Volume of Fluid (VOF) Method to Model Shape Change during Electrodeposition" authored by E. Karimi-Sibaki et al.

The following proposed algorithm designed only for quadrilateral square shaped mesh elements in the deposit region. The normal vector to the interface ($\vec{n} = (n_x, n_y)$) is given by: $\vec{n} = \frac{\vec{\nabla}\beta}{\|\vec{\nabla}\beta\|}$ where β denotes volume fraction of deposit. The interface area (a_{VOF}) is bounded ($a_{VOF} = [0, \sqrt{2}a]$) where a is the side length of the square element. As shown in Figure 1, three situations are plausible. In situation (I), the absolute value of one of the components of the unit normal vector is equal to one ($|n_x| = 1$ or $|n_y| = 1$) where $a_{VOF} = a$. In situation (II), the interface connects two perpendicular sides of the cell so that β is the area of a triangle in 2D. Contrastingly, the interface connects two parallel sides of the cell in situation (III), whereby β is the area of a trapezoid in 2D.

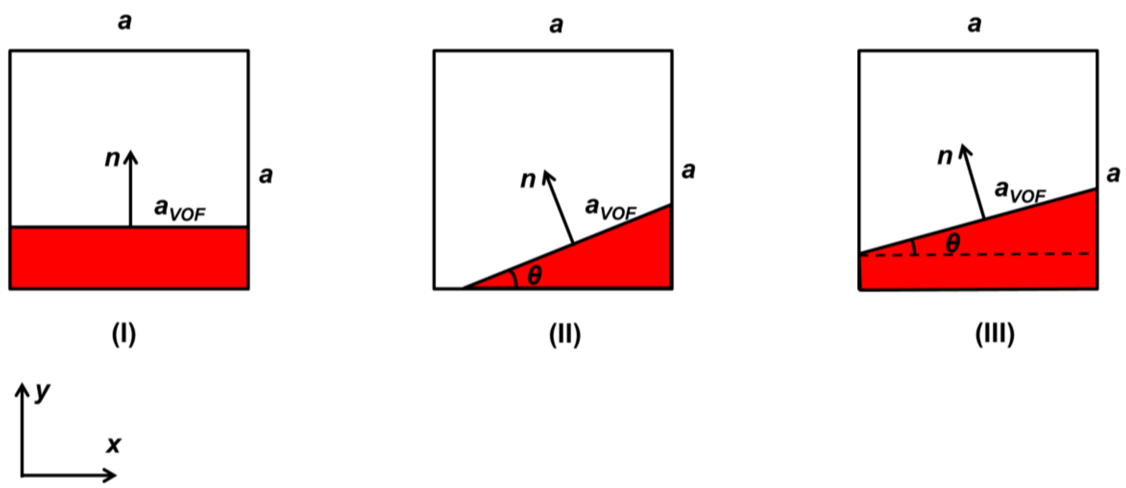

Figure 1. All possible situations (I, II, and III) which are considered to compute the VOF interface area are illustrated. The red zone demonstrates the area filled by deposit.

The geometric definition of angle θ is shown in Figure 2. The angle can be calculated using trigonometric functions and components of the unit normal vector to interface ($\vec{n} = (n_x , n_y)$) as follows: $\tan\theta = \min(\frac{|n_y|}{|n_x|}, \frac{|n_x|}{|n_y|})$ when $\theta \le \frac{\pi}{4}$.

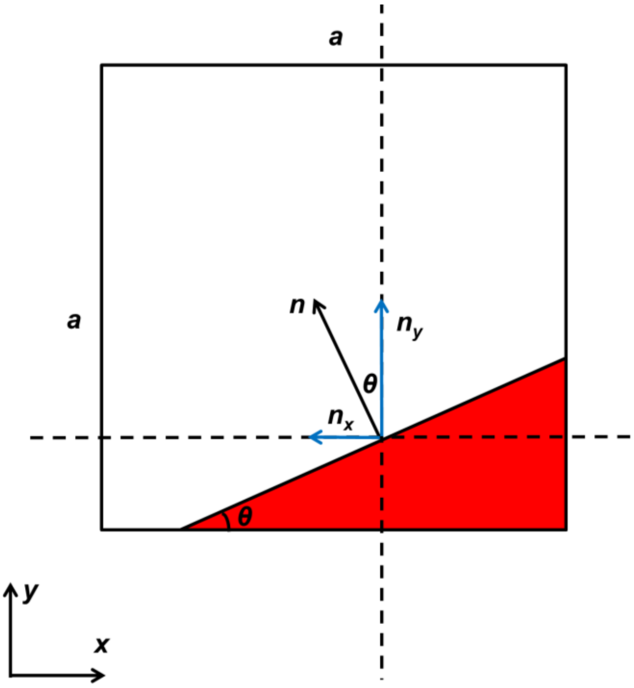

Figure 2. Geometric definition of angle θ.

To distinguish between situation (II) and situation (III) shown in Figure 1, the limit volume fraction (β_L) is calculated. β_L is the value of β when situation (II) transitions to situation (III) as shown in Figure 3,

$$\beta_L = \frac{Triangle\ area}{Square\ area} = \frac{\frac{1}{2}a.a.\tan\theta}{a^2} = \frac{\tan\theta}{2} = \min(\frac{|n_y|}{2|n_x|}, \frac{|n_x|}{2|n_y|}).$$

The sum of green and red zones makes a triangle corresponding to β_L with an angle $\theta \le \frac{\pi}{4}$ which has always one side length of a.

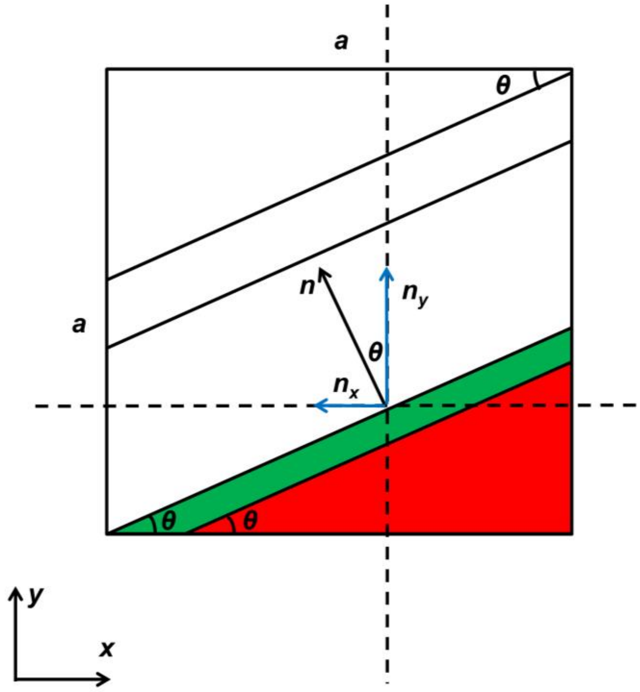

Figure 3. Situation (II) transitions to situation (III). The volume fraction (β) is the red zone. The limit volume fraction (β_L) is the sum of green and red zones.

Situation (II) occurs when ($0 < \beta < \beta_L$) and the volume fraction of deposit is the area of triangle. Then (a_{VOF}) can be easily calculated as follows:

$$a_{VOF} = \sqrt{\frac{2\beta a^2}{|n_x||n_y|}}$$

As shown in Figure 3, situation (II) also occurs when ($\beta_L < \beta < 1$) and the triangle has the volume fraction of (1-β). Then (a_{VOF}) can be easily calculated as follows:

$$a_{VOF} = \sqrt{\frac{2(1-\beta)a^2}{|n_x||n_y|}}$$

In situation (III), the volume fraction of deposit is the area of trapezoid. Not that, in situation (III), a_{VOF} remains constant with the increase of volume fraction ($\beta_L < \beta < 1 - \beta_L$) as long as θ is not changed. Therefore (a_{VOF}) can be calculated as follows:

$$a_{VOF} = \min\left(\frac{a}{|n_x|}, \frac{a}{|n_y|}\right).$$