

Supplementary Information: “Check mass conservation”

This document is a supplementary information for the paper entitled “A Volume of Fluid (VOF) Method to Model Shape Change during Electrodeposition” authored by E. Karimi-Sibaki et al.

As an example, examination of the conservation of mass and charge according to Faraday's laws is discussed using monitored/recorded data from “Multidirectional shape change”.

As described in the manuscript, the mass source term (\dot{m}) determines the local growth of the cathode surface (Eq. 3 in the manuscript),

$$\dot{m} = \frac{a_{VOF} M}{z F V} (\vec{j} \cdot \vec{n}). \quad (1^*)$$

Reformulating the above equation, one can obtain the speed of deposit front (u_{df}) as follows (Subramanian and White 2002):

$$u_{df} = \left(\frac{M}{z F \rho} \right) (\vec{j} \cdot \vec{n}) \quad (2^*)$$

Where M is molecular weight, z is Number of exchanged electrons, F is Faraday constant, and ρ is density. Integral over the surface of deposit front on both sides of Eq. (2*) gives,

$$\text{Added deposit volume} = \left(\frac{M}{z F \rho} \right) * \text{current} \quad (3^*)$$

In Eq. (3*), added deposit volume at each time step and current are continuously monitored/recorded during calculations. Furthermore, we call the constant $\left(\frac{M}{z F \rho} \right)$ the **ratio**. The recorded values of the added deposit volume and current are shown in Figure 1*.

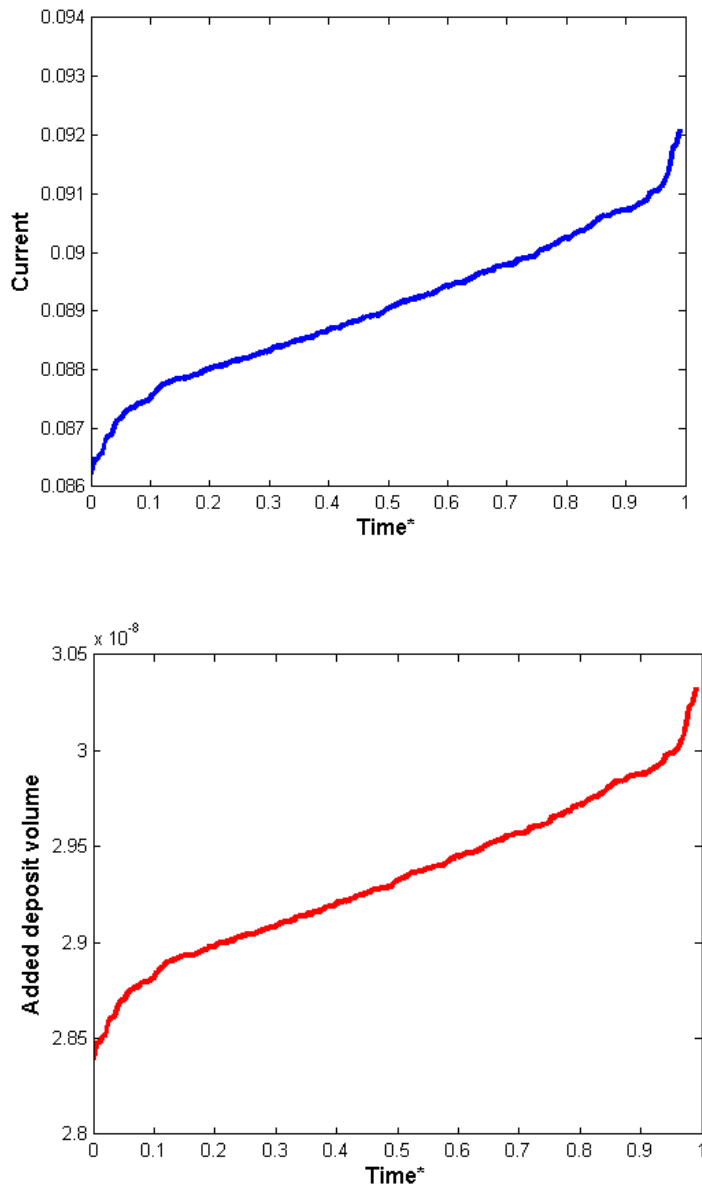


Figure 1*. Recorded values of added deposit volume and current during calculation. Time* is normalized time.

As shown in [Figure 2*](#), and considering [Eq. \(3*\)](#), plotting added deposit volume versus (ratio*current) results in a line with the slope equals to one.

In other words, (more clear way) and considering [Eq. \(3*\)](#), plotting added deposit volume divided by (ratio*current) versus time must be always equal to one as shown in [Figure 3*](#).

This is to ensure us on overall mass and charge conservation.

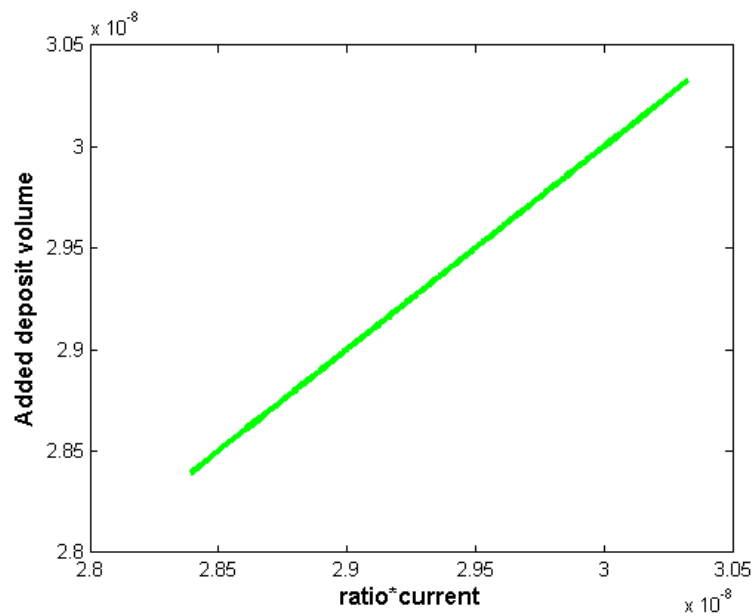


Figure 2*. Added deposit volume versus ratio*current is plotted.

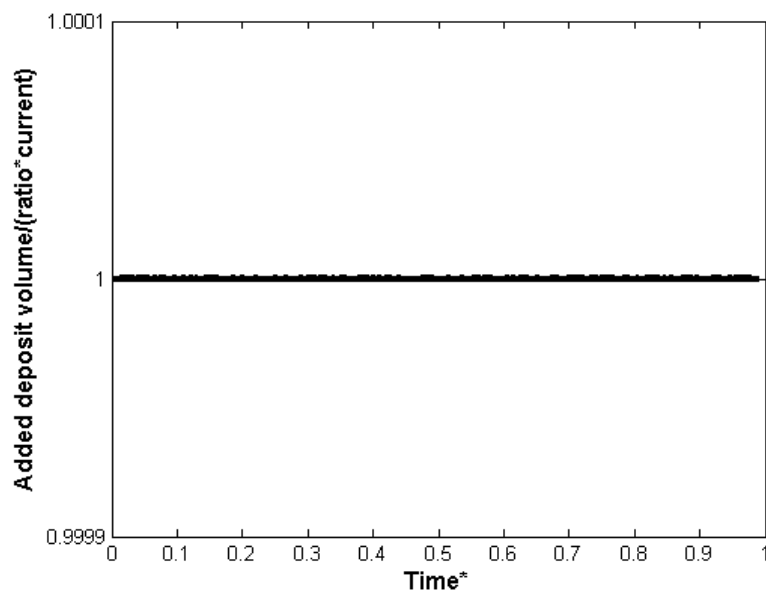


Figure 3*. Added deposit volume/(ratio*current) is plotted against time. Time* is normalized time.